Coastal Engineering Technical Note

DIRECT METHODS FOR CALCULATING WAVELENGTH

PURPOSE: To provide two methods for directly and accurately approximating the wavelength of known period waves in any water depth. Present methods involving the solution of the wave dispersion equation require the use of iterative procedures. Direct approximation using the recommended methods reduces computer time for programs requiring a large number of wavelength calculations, such as those required in wave refraction programs.

GENERAL: The deepwater wave length L_o of a wave is defined by the equation

$$L_o = \frac{gT^2}{2\pi}$$

where T is the wave period and g is gravitational acceleration. Exact computation of the wavelength L in any other water depth d requires the solution of the dispersion equation

$$L = L_o \tanh\left(\frac{2\pi d}{L}\right)$$

which usually requires iterative computations. Values of d/L versus d/L_o are tabulated in Table C-1 of the Shore Protection Manual (SPM) (1984). These tabulated values can be used for single calculations of a particular wavelength. However, the tabulated values are not suitable when computer programs are used for wave refraction, and the wavelength must be determined repetitively by the refraction program.

HUNT'S METHOD: The equation for wavelength can be restated as

$$L = \frac{gT^2}{2\pi} \tanh\left(\frac{2\pi d}{L}\right)$$

To solve for the wavelength, Hunt (1979), and recently, Chen and Thompson (1985) used a Pade′ approximation which gives

U. S. Army Engineer Waterways Experiment Station, Coastal Engineering Research Center
P. O. Box 631, Vicksburg, Mississippi 39180

$$\frac{gT}{2\pi} \tanh \left(\frac{2\pi d}{L}\right) \approx \sqrt{\frac{gd}{F}}$$

where the term $\sqrt{\frac{gd}{F}}$ is an approximation for the wave celerity and

$$F = G + \frac{1}{1.0 + 0.6522G + 0.4622G^2 + 0.0864G^4 + 0.0675G^5}$$

and

$$G = 2\pi \left(\frac{d}{L_o}\right) = \left(\frac{2\pi}{T}\right)^2 \frac{d}{g}$$

The wavelength is then given as

$$L = T\sqrt{\frac{gd}{F}}$$

This gives the wavelength to an accuracy of 0.1 percent. Higher accuracy can be obtained by using a higher order expansion (Hunt 1979).

NIELSEN'S METHOD: Nielsen (1982) gives an explicit formula for L as follows:

$$L = \sqrt{2\pi d L_o} \left(1 - \frac{d}{L_o}\right)$$

This formula gives an accuracy to within 1 percent for $0 \leq \frac{d}{L_o} \leq 0.35$ and becomes increasingly inaccurate at larger depth, i.e., $\frac{d}{L_o} > 0.35$ (Figure 1). It should be noted that Nielsen's method should not be used for deepwater wave computation, and Equation 4-b of the SPM (1984) provides a close approximation for waves in deeper waters (Figure 1).

* * * * * * * * * * * * * * * * * EXAMPLE * * * * * * * * * * * * * * * * * *

GIVEN: A deepwater wave has a period of 16 sec.

FIND: The wavelength in a water depth of 100 ft.

SOLUTION: (a) Using Hunt's method

$$G = \left(\frac{2\pi}{T}\right)^2 \frac{d}{g} = \left(\frac{2\pi}{16}\right)^2 \frac{100}{32.174} = 0.479$$

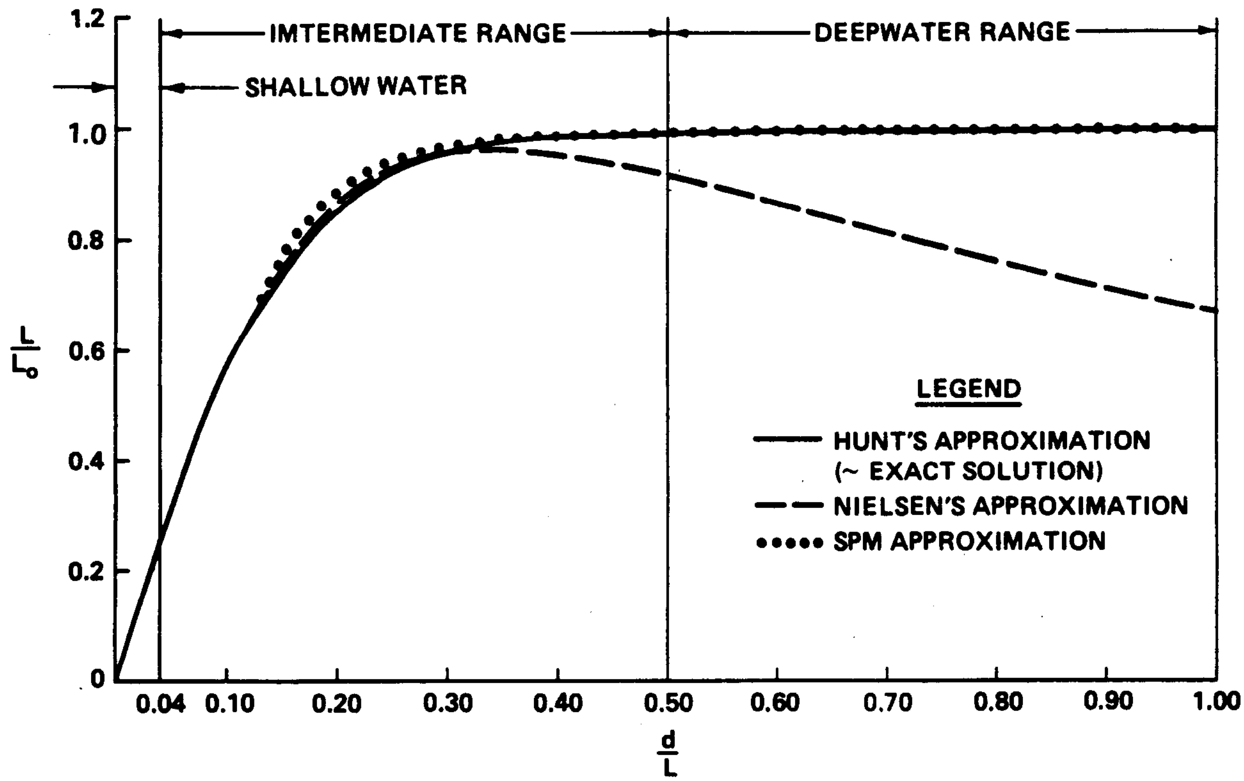

Figure 1. Comparison of approximate solutions for wavelength computations

$$F = G + \frac{1}{1 + 0.6522G + 0.4622G^2 + 0.0864G^4 + 0.0675G^5}$$

$$F = .479 + \frac{1}{1 + 0.6522(0.479) + 0.4622(0.479)^2 + 0.0864(0.479)^4 + 0.0675(0.479)^5}$$

$$= 1.181$$

$$L = T\sqrt{\frac{gd}{F}} = 16\sqrt{\frac{32.174\ (100)}{1.181}} = 835.1 \text{ ft}$$

The exact solution is known to be 834.7 ft. The wavelength obtained using Hunt's approximation has an error on the order of 0.05 percent.

(b) Using Nielsen's method

$$L_o = \frac{gT^2}{2\pi} = 1310.887 \text{ ft}$$

$$L = \sqrt{2\pi d\ L_o}\left(1 - \frac{d}{L_o}\right) = \sqrt{2\pi(100)(1310.887)}\left(1 - \frac{100}{1310.87}\right) = 838.3 \text{ ft}$$

The wavelength obtained using Nielsen's approximation has an error on the order of 0.43 percent.

* *

ADDITIONAL INFORMATION: Contact Dr. Fred Camfield at (601) 634-2012, Coastal Design Branch, CERC.

REFERENCES :

Chen, H. S. and Thompson, E. F. 1985. "Iterative and Pade Solutions for the Water-Wave Dispersion Relation," Miscellaneous Paper CERC-85-4, US Army Engineer Waterways Experiment Station, Vicksburg, Miss.

Hunt, John N. 1979. "Direct Solution of Wave Dispersion Equation," Journal of the Waterways, Port, Coastal and Ocean Division, No. WW4, ASCE, pp 457-459.

Nielsen, P. 1982. "Explicit Formulae for Practical Wave Calculations," Coastal Engineering, Vol 6, No. 4, pp 389-398.

Shore Protection Manual. 1984. 4th ed., 2 vols, US Army Engineer Waterways Experiment Station, Coastal Engineering Research Center, US Government Printing Office, Washington, DC.